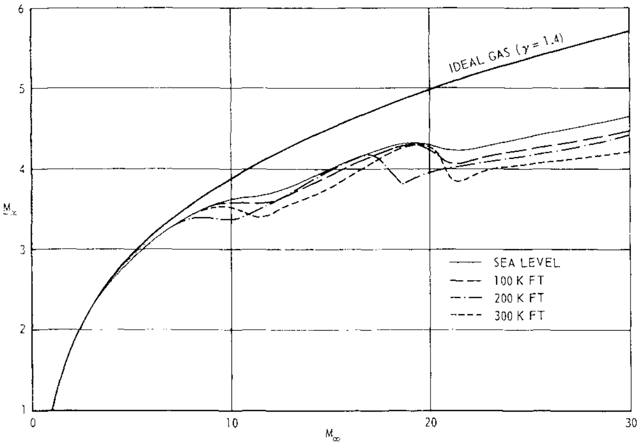

Fig. 1 Asymptotic Mach number for blunt bodies.

atmospheric pressure. Because the flow wetting the body or making up the wake passes through the normal part of the bow shock, a large entropy increase occurs. The static temperature far downstream will then be higher than atmospheric static temperature, and the local velocity will be less than freestream velocity. Thus, the local Mach numbers will be lower than the freestream values.

The asymptotic values of these local "entropy layer" Mach numbers can be determined without knowing the details of the flowfield. Across the shock at the freestream Mach number M_∞, the energy equation holds

$$(u_\infty^2/2) + h_\infty = (u_2^2/2) + h_2 = \text{const} \qquad (1)$$

Because of the increase in entropy, h_2 is greater than h_1, and therefore u_2 is less than u_∞, whereas from the shock-wave solution, P_2 is greater than P_∞. By applying the energy equation between the point just behind the shock and far downstream, where $P_{-\infty} = P_\infty$, and making use of the fact that $(S/R)_{-\infty} = (S/R)_2$, the asymptotic velocity $u_{-\infty}$ and asymptotic speed of sound $a_{-\infty}$ are determined. We then have

$$M_{-\infty} = u_{-\infty}/a_{-\infty} \qquad (2)$$

This asymptotic Mach number is then useful in checking numerical and theoretical results in regions where the local pressure has nearly reached the ambient pressure.

Figure 1 shows the variation of this asymptotic Mach number with freestream Mach number for several altitudes. The atmosphere used was the 1962 standard,[1] and the local velocity and speed of sound were evaluated by means of a computer program for real-gas normal shocks, coupled to a small program that uses the energy equation to determine the asymptotic velocity. The thermodynamic properties were determined by Hansen's method.[2] The first dip in each real-gas curve in Fig. 1 corresponds to completion of the dissociation of the oxygen molecules; the second corresponds to the completion of the dissociation of the nitrogen molecules. For comparison, the ideal gas curve for $\gamma = 1.4$ is included. Note that the first differences between ideal- and real-gas data appear at Mach numbers between 3 and 4.

The data given in Fig. 1 should provide a rapid check on the operation of both equilibrium and nonequilibrium programs in the far downstream region. Significant deviations from these results are indicative of a possible programing error.

References

[1] "U. S. Standard Atmosphere, 1962," U. S. Committee on Extension to Standard Atmospheres, NASA, U. S. Air Force, and U. S. Weather Bureau (1962).

[2] Hansen, C. F., "Approximations for the thermodynamic and transport properties of high-temperature air," NASA TR R-50 (1959).

Comments on "Criteria for Self-Similar Solutions to Radiative Flow"

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In a recent note,[1] Der discussed the similarity criteria for a radiating piston-type problem in the thin and thick limits. This writer wishes to point out that a generalized version of this problem with the thin and thick limits as special cases has been published previously elsewhere.[2] Der's results can be obtained from Eqs. (14a) and (15) of Ref. 2 by setting $\omega = 0$ which corresponds to the case of ambiant density being constant. The symbol n denotes the same in both papers, but β of Ref. (2) corresponds to α in Der's thin criterion and $-\alpha$ in his thick criterion. This is because Der assumed the absorption coefficient as being proportional to $p^\alpha \rho^\beta$ for the thin case and to $p^{-\alpha}\rho^{-\beta}$ for the thick case, whereas this writer considered the same as being proportional to $\rho^\alpha T^\beta$ for arbitrary opacity including the thin and thick limits.

References

[1] Der, J. J., "Criteria for self-similar solutions to radiative flow," AIAA J. 4, 1107–1108 (1966).

[2] Wang, K. C., "The piston problem with thermal radiation," J. Fluid Mech. 20, 447–455 (1964).

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Comment on "On Dynamic Snap Buckling of Shallow Arches"

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An interesting analysis of the title problem using Galerkin's method has been presented by Humphreys. The mathematical and computational features of the analysis are similar to those which arise in several problems concerning the dynamic behavior of plates and shells. Here we wish to discuss briefly the choice of modal functions used in the Galerkin expansion ϕ_m and the evaluation of the integrals of these functions and their derivatives which arise in the analysis.

In Ref. 1, the infinitely wide (or two-dimensional) arch has been treated using the natural modes of simply supported and clamped beams for the ϕ_m. Humphreys notes that although the integrals of ϕ_m for the simply supported case are handled easily, those for the clamped case are difficult to evaluate numerically because of the problem of small differences between large numbers. It is of interest thus to point out that these integrals have been evaluated analytically by Gallagher and Mercer[2] and also by Ketter.[3,4] Gallagher and Mercer do not indicate their method; Ketter, however, has used a clever variation of integration by parts.

The other point to be made is with regard to the finite width (or three-dimensional) problem. Dowell[5] has carried out such an analysis for the simply supported plate (the extension from the plate to a shallow shell is mathematically trivial though physically important) in a different physical

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context. Although the algebraic complexity is increased considerably over that of the infinitely wide plate problem, no essential difficulty arises in the computation. However, with regard to the clamped edge plate, the natural modes are not the most advantageous to use. (The natural modes of a clamped plate are not available in analytical form, of course. Here reference is made to the clamped beam modes.)

As one of the writer's students has discovered, this is because of the necessity of solving an inhomogeneous equation for the stress function F, where the plate deflection W is (temporarily) considered known. In obvious notation (this is one of Von Karman's large deflection equations),

$$\nabla^4 F/Eh = (\partial^2 W/\partial x \partial y)^2 - (\partial^2 W/\partial x^2)(\partial^2 W/\partial y^2) \quad (1)$$

In the usual Galerkin approach, W is expanded as

$$W = \Sigma\Sigma a_{mn}(t)\ \phi_m(x)\phi_n(y) \quad (2)$$

For the natural modes of a clamped beam,

$$\phi_m(x) = \cosh G_m x - \cos G_m x + \alpha_m(\sin G_m x - \sinh G_m x) \quad (3)$$

where G_m and α_m are characteristic constants. This choice of ϕ_m leads to considerable algebraic complexity in solving for F from (1) and performing subsequent integrations. A better alternative is

$$\phi_m(x) = \cos(m-1)\pi x - \cos(m+1)\pi x \quad (4)$$

This expansion has been used previously for linear plate problems[6] and has proven quite satisfactory for the nonlinear problem as well.

References

[1] Humphreys, J. S., "On dynamic snap buckling of shallow arches," AIAA J. **4**, 878–886 (1966).
[2] Gallagher, A. P. and Mercer, A. M., "On the behavior of small disturbances in planel couette flow," J. Fluid Mech. **13**, Pt. 1, 91–100 (May 1962).
[3] Ketter, D. J., "Coupled panel cavity vibrations," AIAA J. **3**, 2164–2166 (1965).
[4] Ketter, D. J., "Addendum: 'Coupled cavity panel vibrations'," AIAA J. **4**, 768 (1966).
[5] Dowell, E. H., "Nonlinear oscillations of a fluttering plate," AIAA Preprint 66-79 (1966); also AIAA J. **4**, 1267–1276 (1966).
[6] Dowell, E. H. and Voss, H. M., "The effect of a cavity on panel vibration," AIAA J. **1**, 476–477 (1963).

Reply by Author to E. H. Dowell

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THE author wishes to thank E. H. Dowell for his useful comments on Ref. 1 regarding clamped beam and panel modes. Since the "bootstrap" technique referred to by Dowell[2] for evaluation of coefficients such as

$$\int_0^\pi \phi_s \phi_n'' d\xi$$

is not explained clearly in his Ref. 3 and appears to be such a powerful shortcut, perhaps a few additional details would be of value. Essentially, the evaluation of integrals such as the preceding can be accomplished by successive partial integration, making use of the cyclic properties of the beam modes in that they satisfy the beam equation $\phi_n'''' = m_n^4 \phi_n$. If sufficient partial integrations are performed, using the beam equation at different times for both ϕ_s and ϕ_n, the original integral is regained eventually, plus several terms to be evaluated at the integration limits. The latter terms are much easier to evaluate than the process performed by the author in Ref. 1 of forcing the integration through via a series expression.

One further point not mentioned in Dowell's Refs. 2 or 3 is that for any mode index higher than unity the multiplicative factor

$$\alpha_n = \frac{\cos m_n\pi - \cosh m_n\pi}{\sin m_n\pi - \sinh m_n\pi}$$

(in Ref. 1 notation) that appears in the mode shapes is 1.00 to at least three significant figures. This further simplifies the expressions for the integral coefficients for almost all the terms needed. Such approximate expressions as derived in Ref. 1 (albeit by a tedious route) are identical with comparable expressions derived from Dowell's references.

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[1] Humphreys, J. S., "On dynamic snap buckling of shallow arches," AIAA J. **4**, 878–886 (1966).
[2] Dowell, E. H., "Comment on 'On dynamic snap buckling of shallow arches'," AIAA J. **4**, 1887–1888 (1966).

Errata: "Electrical Conductivity of Partially Ionized Gases"

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[AIAA J. **4**, 1012–1019 (1966)]

THE following correction should be made to the above article.

1) Page 1014, fourth line in Eq. (12) should read:

$$q^{02} = 4 \sum_{j=3}^{\nu} n_j \left(\frac{35}{4}\ \Omega_{1j}{}^{11}\right) - \ldots$$

2) Page 1015, second column, line 6 should read:

$$\epsilon_m = \tfrac{1}{4}(m^2 - m)\gamma_m$$

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Erratum: "Relationship between Filler Distribution and Uniaxial Rupture of Composite Solid Propellants"

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[AIAA J. **4**, 1044–1049 (1966)]

EQUATION (13) should read

$$\lambda_r = [(\lambda/J_3) - V_f]/(1 - V_f)$$

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